

INSTRUCTIONS: Attempt all problems. Calculators and/or mobile phones are not allowed. Each (sub)question is worth the points indicated on the left.

1. (7 points each) Evaluate each of the following integrals.

(a) $\int \tan^2 x \cos^5 x \, dx$

(b) $\int \sin(\ln x) \, dx$

(c) $\int \frac{dx}{x^2 \sqrt{x^2 + 1}}$

(d) $\int \frac{3x^2 - 4x + 5}{(x - 1)(x^2 + 1)} \, dx$

2. (7 points) Determine whether the improper integral converges or diverges. If it converges, find its value.

$$\int_0^{\infty} x e^{-x} \, dx$$

3. (7 points) Consider the curve

$$x = \frac{2}{9}t^{3/2}, \quad y = \frac{1}{3}t^3, \quad 0 \leq t \leq 2$$

(a) Find the slope of the tangent line to the curve at the point corresponding to $t = 1/4$.

(b) Find the length of the curve.

4. (8 points) Find the area of the region inside both of the curves:

$$r = 1 + \sin \theta, \quad \text{and} \quad r = 3 \sin \theta.$$